Recent developments and challenges ahead in hydrological modelling

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ABSTRACT

Since the introduction of the concept of the hydrologic cycle by Leonardo Da Vinci in the 15th century, several significant developments have taken place in hydrological modelling. Among them in the early days are the rational method, unit hydrograph theory, infiltration theories, extreme value theory, tank model, kinematic wave theory, Stanford watershed model, Xinanjiang model and variable infiltration capacity model. Hydrology has since witnessed an enormous growth in the latter part of the twentieth century as a result of technological and methodological advances. Invention of powerful computers, remote sensors, geographical information systems (GIS), worldwide web and networking have facilitated extensive data collection (both in time and in space), better data sharing, formulation of sophisticated mathematical methods including those for studying the inherent non-linearities, and development of highly complex models. Hydrological modelling is a challenging task nowadays because of the multi-faceted nature of the problem and the various choices available. Despite the proliferation of models and modelling techniques in the past few decades there is no unique approach or model that suits all and all purposes. Reviewing the literature reveals that hydrologic modelling based on the linear and stationary assumptions has reached saturation levels. Advances in analysis of non-linear systems in recent years have made it possible to explore the hydrologic system (which has always been non-linear but for simplicity has been assumed linear) within a non-linear framework. There is also evidence to believe that the stationarity assumption made in the analysis of hydrologic time series in the past is no longer valid due to human interference in the natural hydrologic cycle. In this paper, the author attempts to highlight some criteria for the choice of a model, data issues, modelling issues and limitations of different types of models including the challenges in calibration and uncertainty.

1. INTRODUCTION

Since the introduction of the rational method which perhaps is the first hydrological model to be developed, hydrological modelling has witnessed an enormous growth in the latter part of the twentieth century as a result of technological and methodological advances. Invention of powerful computers, remote sensors, geographical information systems (GIS), and world wide web and networking have facilitated extensive data collection (both in time and in space), better data sharing, formulation of sophisticated mathematical methods (including those for studying the inherent non-linearities, scalings, and thresholds), and development of highly complex models. Such advances have enabled hydrologists to mimic the real hydrologic systems better and more precisely than two to three decades ago. However, the existence of such a plethora of hydrologic models and modeling techniques, with each one having its own pros and cons, can sometimes be confusing to even experienced hydrologists.

Among the developments in hydrological modelling in the pre-computer era are the rational method [1], unit hydrograph theory [2], infiltration theories [3-5], extreme value theory [6], and the kinematic wave theory [7]. Prior to the development of such models, there have been related developments in fluid mechanics such as Chezy's equation (1769), Darcy's equation (1856), Manning's equation (1885) etc. which have also become household names in hydrology. It is also a

fact that fluid mechanics, hydraulics and hydrology have no crisp boundaries. In the pre-computer era, the analogy between water flow and electricity flow have been utilised to construct analogue models (electrical circuits) in which the analogies between electric current and flow rate, electrical resistance and friction, voltage (or potential difference) and pressure, and electrical capacitance and storage have been used. However, such analogue models, despite their simplicity in construction and low cost, have become outdated with the emergence of digital computers.

The developments that have taken place in the computer era can be broadly classified into conceptual models, physics-based models and data driven models. Among the early conceptual models are the Stanford watershed model [8], Xinanjiang model [9-11], Tank model [12], HEC series [13,14], linear cascade model [15-18], linear channel [19], and the variable infiltration capacity (VIC) model [20]. Physics-based models start with governing equations based on the laws of conservation of mass, momentum and energy (St.Venant's equations). They are process-based but require a great deal of assumptions and simplifications, initial and boundary conditions, solution domain discretization, solution schemes to solve the resulting governing equations numerically, and parameter identification, calibration and validation. Solution schemes usually employ the finite difference methods, finite element methods and their combinations. Most such 'physics-based' models make approximations and assumptions making them over-simplified and losing the true meaning of physics-based models.

More recently, many types of data driven models have emerged and received attention and acceptance. Among the different types are regression models, stochastic models, artificial neural networks, support vector machines, genetic algorithms and genetic programming, fuzzy logic models, neuro-fuzzy models, and dynamical systems approach type models which makes use of theory of chaos. Data driven models are attractive from a number of points of view. For example, data contain all the measurable information about the system; they are easier to formulate and interpret; they are the only option when other approaches are infeasible; there is no need for *a priori* understanding of the processes involved; and particularly suited to theory weak data rich situations. The objective of this paper is to describe briefly the criteria for the choice of a model, the challenges that lie ahead in the implementation of a model, and to highlight some of the recent data driven types of modelling including their applications.

2. CRITERIA FOR THE CHOICE OF A MODEL

One of the guiding principles of any modeling attempt is that it should be useful to solve or understand a particular problem under a given set conditions and constraints. In any exercise aimed at developing a model, there is a benefit to be expected and an associated cost. A reasonable balance between the cost and the benefit should be sought to justify the attempt. Driven by the pressure to publish, many "new" models and modeling techniques appear in the hydrologic literature, a majority of which happen to be adding only marginal value to existing knowledge, usually at an unjustifiable cost. There is also the unfortunate fact that research is resources-driven rather than needs driven. There is also the perception that complex models are for the developed world whereas simple models are for the developing world. The question then is, what criteria should be used in selecting or developing a model that suits a particular need under a given set of conditions and constraints. The starting point should be to decide whether the model is for a practical purpose to solve a particular problem, or for an academic purpose with a view to better understand the hydrologic system. The views are divided. There is a school of thought that advocates the principle that better understanding of the system is more important than the end result. There is also the other school of thought that advocates the principle that it is the end result that matters and not how it is obtained. In developed countries, where relatively more resources are available for research, the approach adopted has been to explore the hydrologic system in a distributed or semi-distributed manner. It has advantages and disadvantages. The advantages are mainly of a potential nature, meaning that it is only when all the components that constitute the model are known, or can be known, a priori, there will be better understanding of the system. This condition rarely exists in the real world. On the other hand, in less developed countries, where there are severe constraints in resources for research, the

approach adopted is to look for simple, practical, and result-oriented methods that would suit the problem.

3. CHALLENGES AHEAD

3.1 Data issues.

The accuracy and reliability of the outcome of a model depends upon the accuracy and reliability of the data used as inputs. For simple hydrologic models, the basic input is the rainfall, which varies spatially and temporally. Present day rain gages can measure rainfall to a very high degree of accuracy, but a reasonable spatial and temporal resolution is necessary to ensure that the data are representative. Averaging out the data has the tendency to smooth out variations, thereby distorting the real situation. A compromise is often needed to strike a balance between the resources available and the accuracy of the expected result. The second most important hydrologic variable for modeling is the discharge resulting from rainfall, which can be considered as an integrator of all catchmentscale processes. Direct measurements of discharges are rarely made under normal conditions. They are derived from stage measurements using rating curves. Stage measurements can be made quite precisely, but the rating curves depend upon many factors, such as the techniques and instruments used to measure velocities and channel hydraulic parameters, and whether or not measurements cover the entire range of possible values. Very often, rating curves are established under normal-flow conditions, and extrapolated to obtain discharges at high-flow conditions, thereby introducing an uncertain error. Measurements at high-flow conditions are usually not made, because they are difficult, dangerous, and costly. There are other hydrologic processes, such as evaporation and evapotranspiration, infiltration, interception, and depression storage, that contribute to the basin-scale hydrologic system, and their inclusion requires some approximations and assumptions while their exclusion results in over-simplification. Another factor that contributes to the uncertainty is the noise that is inherently present in all types of measured data. In addition to hydrologic data, geometric, topographic, geologic, and land use data are needed for distributed type of models. On a local scale, such data can be found in limited situations. The resolutions vary and depend upon the region and the catchment. On a global or regional scale, remotely-sensed topographic data are available, particularly from satellite observations. Their resolutions also vary, but the publicly available data sets do rarely have resolutions finer than 1 km x 1 km horizontally, and a few 10's of meters vertically. The results of any distributed model that uses such coarse data will have inherent errors of the same order or higher, than those of the input topographic data.

3.2 Modeling issues.

Hydrologic models can be classified according to several different criteria. On a broad basis, they can be classified as data-driven and physics-based. The former type includes all models that do not consider the physics of the transformation of rainfall to discharges, whereas the latter type, in principle, considers laws of physics in the modeling process. Data-driven models are relatively easier to implement, but not without problems. Physics-based models consider the catchment processes from a physics point of view, but their formulation, calibration, and implementation are guite resource and expertise demanding. Their problems are also of a higher magnitude. So far, no fully physics-based model has been successfully applied to a catchment, without making drastic assumptions and simplifications. There are also conceptual semi-distributed models that attempt to lump system characteristics on a small scale. The main challenges that lie ahead in the data-driven modelling front include choosing between stochastic and deterministic approaches, lumped and semi-distributed approaches, linear and non-linear approaches, and stationary and non-stationary assumptions. The choice depends upon the purpose. Most data-driven models are lumped. Linear assumption makes subsequent analysis and application simple but, in many instances, it is far from reality. Non-linear assumption makes the problem more realistic, but at a cost and lacks generality. Similarly, stationarity assumption makes analysis and application simpler, but with human influence (such as climate

change) in the hydrologic system, the stationarity assumption no longer holds in many situations. The next modelling challenge comes from scale issues. If the physics-based distributed approach is to be followed, it is necessary to define a set of governing equations. These include the Saint-Venant equations for overland flow, Richards equation for soil water flow, diffusion-type equation based on Darcy's law and continuity for groundwater flow, Green and Ampt-, Horton-, and Philip-type equations for infiltration, and mass transfer-, aerodynamic-, or combination-type equations for evaporation. Most such equations have been derived for a continuum, and whether they are valid in the scale of typical distributed models is an unresolved issue.

3.3 Parameters and their calibration issues.

All models need calibration before they could be applied. The normal practice is to compare the outcome of the model to the expected outcome and adjust the parameters using some optimization algorithm until the cumulative difference between them, as defined by an objective function, is a minimum. For models with a small number of parameters, this is not difficult. However, as the number of parameters in the model increases, the problem of finding a global minimum of the objective function becomes difficult. The objective function often gets trapped at a local minimum. To deal with the problem of multiple local optima, global search methods, in which a parallel search of the solution space (as opposed to a point by point serial search) by using a population of potential solutions, is used. The capability of such techniques for effective "exploration" of the search space makes them less susceptible to get trapped in a local optima. Popular global search methods include populationevolution-based search strategies, such as the shuffled complex evolution (SCE) algorithm [21] and genetic algorithm (GA) [22]. In the early stages, optimization methods focused mainly on the selection of a single-objective measure of the distance between the model-simulated output and the measured data and the selection of an automatic optimization algorithm to search for the parameter values which minimize that distance [23]. In recent years, because of the increase in the availability of measured data, multi-objective optimization methods have received more attention, as many of the recent distributed models simulate several watershed output fluxes (e.g. water flux, energy flux, sediment flux, water quality indicators) at multiple locations. Even with a single flux, it would be useful to investigate different properties of the flux using multi-objective functions or different combinations of the transformation of the original flux. Based on the original SCE algorithm, recent studies have led to the development of the shuffled complex evolution Metropolis (SCEM) and the multi-objective shuffled complex evolution Metropolis (MOSCEM) algorithms [24,25]. Direct comparison of these methods would be helpful in selecting the most suitable calibration algorithm from the extensively used shuffled complex evolution family of algorithms. For physics-based models, which are necessarily of a distributed nature, use of optimization techniques for calibration defeats the purpose. By definition, the parameters of physics-based models are physically identifiable and thus measurable, at least in theory. In practice, however, such an exercise is not easy to implement, particularly when the catchment characteristics are heterogeneous. No distributed model, which accounts for catchment heterogeneities and spatially-varying hydrologic inputs, that has been calibrated using field measured parameter values exists at the present time. Instead, what is often done is calibrating the parameters of the model using some kind of optimization technique against a single site measured single output data. As a result, most models that start with laws of physics end up as data-driven models, thereby defeating the very purpose of adopting such an approach. Assuming that the above is the only currently available option for calibrating distributed models, the next issue is the choice of the optimization algorithm. In addition to the problem of getting trapped at a local optimum, another problem in multi-parameter optimization is that of equi-finality - a concept originated in the general systems model of Bertalanffy [26], meaning that the same final result may be arrived from different initial conditions and in different ways. In open systems, the final state can be reached by many different ways, whereas in a closed system the equi-finality principle states that there is a cause-effect relationship between the initial state and the final state. In the context of multi-parameter optimization, what this means is that there is no unique set of parameter values, but rather a feasible parameter space from which a Pareto set of solutions is sought. The multi-objective shuffled complex evolution

Metropolis (MOSCEM) algorithms [24,25], which use an improved concept of Pareto dominance by adding the Pareto rank of each of the members of dominated set, is reported to converge towards an optimal Pareto set of solutions. Pareto set of solutions represent tradeoffs with the property that moving from one solution to another results in the improvement of one objective while causing deterioration in one or more others. A state *A* (a set of target parameters) is said to be Pareto optimal if there is no other state *B* dominating the state *A* with respect to a set of objective functions. A state *A* dominates a state *B*, if *A* is better than *B* in at least one objective function and not worse with respect to all other objective functions. The Pareto set represents the minimum uncertainty that can be achieved for the parameters via calibration.

4. RECENT ADVANCES

Recent advances in data driven approaches of hydrological modelling include different types of artificial neural networks such as multi-layer perceptron, radial basis functions, recurrent neural networks, wavelet neural networks and product unit neural networks, support vector machines that can be used for classification as well as regression, dynamical systems approach which can be used for short-time forecasting, genetic algorithms and genetic programming, fuzzy logic approach and neuro-fuzzy approach. Two of the above approaches are highlighted here.

4.1 Artificial neural networks

Artificial neural networks, or ANN's, emulate the brain which can be considered as a biological neural network. The processors operate on the data received via the connections. The transformation of an input to a corresponding output by a single neuron is relatively simple. The complexity and the power of ANN's arise as a result of the interactions of many neurons. A system in which a large number of neurons are interconnected with exposure to the external environment is called an artificial neural network. Very often the neurons are arranged in layers with each layer having a number of neurons.

An artificial neuron can be thought of as a mathematical model of the biological neuron. It has five components: input(s), connection weights, threshold (or bias), activation function(s) and output(s). It receives one or more inputs (analogous to dendrites) from the external environment and sums them up to produce an output analogous to the axon of a biological neuron. This output then acts as the input to the next layer via a synapse. In the same way as a biological neural network has interconnected neurons, artificial neurons also have connections to other neurons. It is also possible for the output from the first layer to exit as the final output.

There are many types of ANN's. For example, multi-layer perceptron (MLP) type, radial basis function type, recurrent type, wavelet type, product unit type etc. The widely used type is the multi-layer perceptron which normally has 3 layers, an input layer, a hidden layer and an output layer. In a multi-hidden layer perceptron the inputs are fully connected to the first hidden layer, each hidden layer is fully connected to the next, and the last hidden layer is fully connected to the output layer.

The main problem in MLP function approximation is how to determine the number of nodes (neurons) in the hidden layer. Too few nodes may not model the process adequately and too many will require a long computational time as well as resulting in over-fitting. The objective of ANN is to model the signal, but over-fitting will fit to the noise as well producing a very good fit which lacks generalization properties when presented with unseen data. The components of an ANN are illustrated in Figs. 1-3. The optimal number of nodes in the hidden layer is determined by cross validation (Fig. 4) using different numbers of nodes which is a trial and error approach. The widely used activation functions are of the sigmoid type and hyperbolic tangent type as defined below:

Sigmoid:
$$f(x) = \frac{1}{1 + e^{-rx}}$$
; hyperbolic tangent, tanh: $f(x) = tanh(rx) = \frac{1 - e^{-rx}}{1 + e^{rx}}$

where *r* is the steepness parameter.







Fig. 2: A typical 3-layer perceptron



Fig 3: Sigmoid activation function



Fig. 4: Illustration of cross validation

4.1.1. An example application

ANN's have been used in many types of hydrological applications. Publications are too numerous to list. A recent application for water level prediction in Surma River in Bangladesh using radial basis function type neural networks is illustrated in Fig. 5. In this application ANN's have been used to forecast daily water levels at the Sylhet gauging station (Latitude: 24° 42'N; Longitude: 91° 53'E) across Surma River which is one of the principal rivers originating from the Assam and Meghalaya hilly areas of India. The network used is a 3-layer MLP with back-propagation algorithm to adjust connection weights. The input layer has 4 nodes representing the water level on the previous day, rainfalls on the same day and 2 preceding days. The output layer gives the current water level. The data used for training covers the period from August 20, 1980 to December 11, 1989, for validation, the period from December 23, 1989 to April 15, 1999, and for application, the period from April 27, 1999 to August 17, 2008. More details of this application can be found in another joint publication by the author [27].



Fig. 5: Application to Surma River, Bangladesh

4.2 Fuzzy Logic Approach

Traditional logic theory involves reasoning based on binary sets which have two valued logic, true or false, yes or no, zero or one. In real life however, much of the information that we come across and process is not so crispy but involves some degree of fuzziness. The truth value may range between the completely true value and the completely false value, leading to a partial truth. The key idea in fuzzy systems is to allow a partial truth to prevail which can be numerically described by a specific function, referred to as the membership function that takes values between 0 and 1. Fuzzy logic enables embedding uncertain or imprecise reasoning in everyday life to computers which operate in exact deterministic ways. Fuzzy logic models are conceptually easy to understand, flexible, tolerant to imprecise data and can handle nonlinear functions of arbitrary complexity and built on the experience of experts. They translate imprecise linguistic information sets into computer usable numerical language.

The general structure of a fuzzy logic system is illustrated in Fig. 6. It consists of a knowledge base which includes a data base and a rule base, and 3 layers of information processing between the external input and output data. The main problems in building fuzzy systems include the selection of the relevant input and output variables, choice of the possible term sets for each linguistic variable, choice of the type of membership functions, fuzzification of the crisp input and output variables, derivation of the rule set, aggregation of the outcomes of the rules and de-fuzzification.



Fig. 6: Structure of a fuzzy logic system

In the hydrological field, applications of fuzzy logic include water level forecasting [28-30], flood forecasting [31,32], infiltration modelling [33], rainfall-runoff modelling [34,35], hydrological time series modelling [36], river discharge prediction using neuro-fuzzy and adaptive neuro- fuzzy inference systems (ANFIS) [37], amongst others. The results of an example application for predicting daily discharges at the Glencourse hydrometric station (6° 58' 33.64" N; 80° 11' 58.71" E) across Kelani River in Sri Lanka using time-lagged upstream discharges and rainfall data for the period 1993-2008 is given below (Fig. 7). More relevant details can be found in separate publications by the author [38,39].



Fig. 7: Discharge prediction using fuzzy logic approach

5. CONCLUDING REMARKS

The challenges in hydrological modelling, despite the present abundance of hydrologic models and modeling techniques, arise as a result of the inadequacy of resources for research, lack of relevant data, lack of expertise, and lack of a clear understanding of the driving force for any hydrologic modeling attempt. In the first place, the choice needs to be based on whether the attempt is needs-driven or resources-driven. When it is needs driven, simple models are adequate, given the limitations arising from data inaccuracy. When it is resources-driven, consideration should be given to the marginal potential benefit that may be accrued against the costs associated with uncertainties and inaccuracies of the data, model formulation, and calibration issues. It is also important to bear in mind the advances in analysis of non-linear systems in recent years which have made it possible to explore the hydrologic system (which has always been non-linear but for simplicity has been assumed linear) within a non-linear framework. There is also evidence to believe that the stationarity assumption made in the analysis of hydrologic time series in the past is no longer valid due to human interference in the natural hydrologic cycle. Any future data-driven approach of hydrologic modelling should take into account this shift in paradium and consider the vital role that non-linear non-stationary dynamics and scaling theories can play. Calibration of parameters and uncertainty analysis are other areas that need attention.

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